BRST QUANTIZATION OF NON-ABELIAN BF TOPOLOGICAL THEORIES

M. I. Caicedo, R. Gianvittorio, A. Restuccia and J. Stephany

Universidad Simón Bolívar, Departamento de Física
Apartado postal 89000, Caracas 1080-A, Venezuela.
e-mail: mcaicedo@usb.ve, ritagian@usb.ve, arestu@usb.ve, stephany@usb.ve

Abstract

The off-shell nilpotent BRST charge and the BRST invariant effective action for non-abelian BF topological theories over D-dimensional manifolds are explicitly constructed. These theories have the feature of being reducible with exactly D-3 stages of reducibility. The adequate extended phase space including the different levels of ghosts for ghosts is explicitly obtained. Using the structure of the resulting BRST charge we show that for topological BF theories the semi-classical approximation completely describes the quantum theory. The independence of the partition function on the metric also follows from our explicit construction in a straightforward way.

BF Topological actions which were introduced in [1,2] as generalizations of three dimensional Chern Simons theories, can also be regarded as a zero coupling limit of Yang-Mills theories. Some results concerning their quantization in the abelian case are fairly well understood, indeed, it has been shown that the partition function for the abelian case may be written in terms of the Ray Singer torsion [3] while other observables such as the Wilson surfaces determine linking and intersection numbers of manifolds in any dimensions.

For the non-abelian case, the metric independence of the quantum BF theory was first proposed in [3] and proved later on in references [4-6] in a direct way. A solution of the master equation of the Batalin-Vilkoviski (BV) approach for the BF action was also presented in [7]. The authors of reference [8] were able to build an off-shell nilpotent Becchi-Rouet-Stora-Tyutin (BRST) operator using an approach similar to the BV procedure but with a Slavnov identity playing the role of the BV master equation. The construction presented in [8] was performed in a covariant gauge and enabled the authors to show that perturbatively the 4 and 5 dimensional theories are anomaly free and finite.

The construction of an off-shell nilpotent BRST charge for a non abelian BF theory based on a canonical (symplectic) formalism was first carried out for the four dimensional model in reference [9]. This operator was built following a modified BFV method introduced earlier in refs. [10] and [11], which simplifies some aspects of the usual approach [12]. Several issues concerning the general structure of of the off shell BRST invariant action for topological theories and in particular a general proof of the metric independence of the partition function were also commented in [9].

In the present paper we explicitly construct the off shell BRST invariant effective action for the non abelian BF theories in any dimension and for any admissible gauge fixing condition. Using this effective action, we prove that in any BF theory the partition function is independent of the gauge coupling constant (e^2), a feature which also appears in Witten's Topological Field Theory [13-16], and which has been recently used to obtain new topological invariants for four dimensional manifolds. It is interesting to notice that Seiberg and Wittenś [17] new approach is based on the structure of N=2 supersymmetric theories in the strong coupling regime and on the independence of the topological theory in the gauge coupling constant properties which are also present in BF theories.

The metric independence of the partition function for these systems arises from general arguments. Although metric dependent terms may appear in the effective action through the gauge fixing conditions, the independence of the partition function on them and consequently on the metric is guaranteed by virtue of the BFV theorem [10-12].

The main difficulties in working with BF actions are related to the fact that they are reducible theories, this property means that many levels of ghosts for ghosts are needed in order to build the proper extended phase space. The BF actions defined on D-dimensional manifolds have exactly D-3 levels of reducibility [3] which make them particularly interesting since such property turns them into explicit examples of reducible theories with a higher (≥ 2) but finite number of stages of reducibility.

The action of a BF theory on a principal bundle with a D dimensional base manifold is given in terms of the curvature 2 form ($F \equiv [\nabla, \nabla]$) and a Lie algebra valued D-2 form (B) as follows:

$$S=\frac{1}{4}\int Tr(B\wedge F)=\frac{1}{4}\int d^Dx\ \epsilon^{\mu_1\cdots\mu_D}(B^a_{\mu_1\cdots\mu_{D-2}})(F^a_{\mu_{D-1}\mu_D}). \tag{1}$$

For the sake of clarity we will begin by discussing the five dimensional case since it presents some features that are absent in four dimensions and which constitute the clue for the discussion of theories formulated in higher dimensions ($D\geq 5$) According to (1), the 5 - dimensional BF theory is given by

$$S=\frac{1}{4}\int d^5x\ \epsilon^{\mu\nu\alpha\rho\sigma}(B^a_{\mu\nu\alpha})(F^a_{\rho\sigma}). \tag{2}$$

After integration by parts and defining $\epsilon^{ijk}\equiv\epsilon^{0ijk}$, the original action (2) may be rewritten as

$$S=\int d^5x\ \dot{A}^a_i[\frac{1}{2}\epsilon^{ijkl}(B^a_{jkl})]+A^a_0\nabla_i[\frac{1}{2}\epsilon^{ijkl}(B^a_{jkl})]+B^a_{0ij}(\frac{1}{2}\epsilon^{ijkl}F^a_{kl}). \tag{3}$$

This expression can be clearly recognized as a canonical action for the simplectic pairs (A^a_i,π^{ia}) where

$$\pi^{ia}=\frac{1}{2}\epsilon^{ijkl}(B^a_{jkl}).$$

The action (3) has a vanishing Hamiltonian, and is subjected to the following set of primary constraints:

$$\phi^a\equiv(\nabla_i\pi^i)^a=\partial_i\pi^{ia}+(A_i\times\pi^i)^a=\partial_i\pi^{ia}+f^{abc}A^b_i\pi^{ic}=0, \tag{4a}$$
$$\Phi^{ija}\equiv\epsilon^{ijkl}(F^a_{kl})=0, \tag{4b}$$

whose associated Lagrange multipliers are A^a_0 and B^a_{0ij}.

The Poisson bracket algebra of the above constraints is given by:

$$\{\phi^a(x),\phi^b(x')\}=f^{abc}\phi^c(x)\delta^3(x-x'), \tag{5a}$$
$$\{\Phi^{ija}(x),\Phi^{klb}(x')\}=0, \tag{5b}$$
$$\{\phi^a(x),\Phi^{ijb}(x')\}=f^{abc}\Phi^{ijc}(x)\delta^3(x-x'), \tag{5c}$$

showing that they are first class. It is important to notice that the set of scalar constraints $\phi^a(x)$ constitutes a closed irreducible sub-algebra of the whole set of constraints while the remaining constraints, namely the tensor ones $\Phi^{ijc}(x)$ are linearly dependent i.e. reducible, since they satisfy the identity:

$$(\delta^m_{[i}\nabla_{j]}\Phi^{ij})^a=0. \tag{6}$$

Moreover, this particular algebra is two times reducible since the matrix $a^{(1)}\equiv a^{(1)ma}_{ij}=\delta^m_{[i}\nabla^a_{j]}$ is itself reducible through the action of the following operator:

$$a^{(2)}\equiv a^{(2)a}_i=\nabla^a_i. \tag{7}$$

This second stage of reducibility holds only on-shell. Indeed, one can easily check that:

$$a_m^{(2)a} a_{ij}^{(1)mb} = \nabla_m^a \delta_{[i}^m \nabla_{j]}^b = \nabla_{[i}^a \nabla_{j]}^b \sim f^{abc}\Phi^{ijc} \approx 0. \tag{8}$$

This last feature will be relevant when calculating the BRST charge (Ω), since in order to have a manifest BRST invariant quantum theory Ω must be off shell nilpotent.

In the modified BFV approach we are using, the extended phase space of a reducible sistem is divided in a minimal sector of canonical pairs with canonical BRST transformation laws and a non-minimal sector. The minimal sector is composed by the original fields (q, p) and a chain of as many pairs of conjugate ghosts and ghosts for ghosts as levels of reducibility. The non minimal sector is composed by extra ghosts, anti-ghosts and Lagrange multipliers whose transformation laws are deviced in a way that ensure the off-shell closure of the BRST transformation [10-12].

For this system, due to the fact that it has two stages of reducibility, the minimal sector of the phase space is composed by the following canonical pairs:

The original fields

$$(A_i^a, \pi^{ia}), \tag{9a}$$

the ghosts associated with the irreducible constraints and their momenta

$$(C_1^a, \mu^{1a}), \tag{9b}$$

and finally, the tower of ghosts and ghosts for ghosts associated to the reducible constraints and their corresponding conjugate momenta

$$(C_{1ij}^{(0)a}, \mu_{(0)}^{1ija}); (C_{11i}^{(1)a}, \mu_{(1)}^{11ia}), (C_{111}^{(2)a}, \mu_{(2)}^{111a}). \tag{9c}$$

Here and in the rest of this paper an embraced super(sub)script indicates the level of reducibility to which a field is associated. Fields associated to irreducible constraints carry no such label. The non minimal sector is discused below.

The BRST charge is constructed using only the variables of the minimal sector. A first candidate for the BRST charge is given by:

$$\begin{aligned}\Omega^{(naive)} =< &C_1\phi - \frac{1}{2}C_1(C_1\times\mu^1)\\ &+ C_{1ij}^{(0)}\Phi^{1ij} + C_{11m}^{(1)}\delta_{[i}^m\nabla_{j]}\mu_{(0)}^{1ij} + C_{111}^{(2)}\nabla_j\mu_{(1)}^{11j}\\ &- C_1(C_{1ij}^{(0)}\times\mu_{(0)}^{1ij}) - C_1(C_{11m}^{(1)}\times\mu_{(1)}^{11m}) - C_1(C_{111}^{(2)}\times\mu_{(2)}^{111}) > .\end{aligned} \tag{10}$$

where $C_1(C_1\times\mu^1) \equiv f^{abc}C_1^aC_1^b\mu^{1c}$,etc. and $< \cdots >$ stands for integration on the space like continuous indexes. This charge has the same basic structure discussed in reference [9] on which we comment briefly. The first line in $\Omega^{(naive)}$ looks like in standard Yang-Mills. This is not surprising since it corresponds to the irreducible sub-algebra (5a). The second line comprises terms coming from the reducible sector of the constraints (4b) and finally, the last line, which is again Yang-Mills like, is due to those Poisson

brackets that mix both sectors of the algebra (5c). This structure looks sufficiently rich for obtaining a nilpotent BRST charge but the direct computation results in

$$\{\Omega^{(naive)},\Omega^{(naive)}\} \sim \{C^{(1)}_{11m}\delta^m_{[i}\nabla_{j]}\mu^{1ij}_{(0)}, C^{(2)}_{111}\nabla_j\mu^{11j}_{(1)}\} \approx \Phi_{ij}C^{(2)}_{111}\mu^{1ij}_{(0)}, \tag{11}$$

obviously meaning that the naive BRST charge is not off-shell nilpotent (a behavior that is not found in D=4) hence, we need terms of higher order in the ghosts to annihilate the unwanted term (11). At this stage, the only contribution that we can add which is compatible with both the ghost and geometrical structure of the minimal sector of the extended phase space is given by the following trilinear combination of $C's$ and $\mu's$: $C^{(2)}_{111}(\mu^{1ij}_{(0)}\times\mu^{1kl}_{(0)})\epsilon_{ijkl}$ according to which the Ω generator should be written as

$$\Omega = \Omega^{(naive)} + \alpha C^{(2)}_{111}(\mu^{1ij}_{(0)}\times\mu^{1kl}_{(0)})\epsilon_{ijkl}, \tag{12}$$

where α must be calculated to ensure the off-shell closure of the BRST algebra. One can in fact do this and obtain

$$\alpha = -\frac{1}{8}. \tag{13}$$

From formulas (8) and (10) and according to comments given above, one realizes that the new term in the Ω generator comes from the fact that the second reducibility condition holds only on-shell ($a^{(2)}a^{(1)} \sim \Phi \approx 0$). We will see that in the higher dimensional case this feature reappears producing a series of polynomial terms in the BRST charge which are generalizations of the trilinear term appearing in (12). There are two important points to remark about the BRST charge that we have just calculated: the first is the fact that (12) is nilpotent off-shell, the second is that Ω is linear in all the ghosts associated to the reducible sector, (i.e. in: $C^{(0)}_{1ij}, C^{(1)}_{11i}$ and $C^{(2)}_{111}$). This latest fact (which generalizes for $D\geq 6$) is the key ingredient in the proof of the independence of the partition function on the gauge coupling constant.

Let us now consider the non-minimal sector of the phase space, the auxiliary fields belonging to this set are necessary in order to build the effective action for the theory and have been already described in reference [10]. These fields include extra ghosts, antighosts and Lagrange multipliers all of which are Lie algebra valued.

In first place in table 1 we will introduce the complete set of auxiliary fields associated with the irreducible constraints (4a)

(C_1,μ^1)	C_2	C_3
λ_1	θ_1	
λ_{11}	λ_{12}	λ_{13}

Table 1: Full set of irreducible auxiliary fields for the $D=5$ BF theory.

In second place we introduce the set of auxiliary fields associated to the reducible constraints, which as in the former case consists of two sectors: one containig C and μ fields and one containing extra lagrange mupltipliers: the λ and θ fields. The structure of the complete (both minimal and nonminimal) set of C fields can be graphically organized in a tree like diagram one of whose branches we show in figure 1.

$$
\begin{array}{ccccccccc}
C_N^{(0)} & & & & & & & & \\
\downarrow & & & & & & & & \\
C_{N1}^{(1)} & \longrightarrow & & \longrightarrow & & \longrightarrow & C_{N2}^{(1)} & \longrightarrow & C_{N3}^{(1)} \\
\downarrow & & & & & & \downarrow & & \downarrow \\
C_{N11}^{(2)} & \longrightarrow & C_{N12}^{(2)} & \longrightarrow & C_{N13}^{(2)} & & \vdots & & \vdots
\end{array}
$$

Figure 1: Reducible C fields for the $D = 5$ Topological theory. $N = 1, 2, 3$

In the diagram above one has to remember that the objects in the minimal sector i.e. $C_1^{(1)}$, $C_{11}^{(1)}$ and $C_{111}^{(2)}$ must be accompanied by their conjugate momenta. In this notation and according to [10], the bracketed superscripts constitute a bookkeeping device that tells to which stage of reducibility does a particular object belong.

The λ and θ fields are many more than the former C fields, nevertheless they may also be organized as a family of layers corresponding to each level of reducibility. For these fields one needs an extra superindex which labels the origin of the object. The complete family is displayed in figure 2 below.

$$
\begin{array}{ccc}
\lambda_1^{0(0)}, \theta_1^{0(0)} & \lambda_{1M}^{0(1)}, \theta_{1M}^{0(1)} & \lambda_{1MN}^{0(2)}, \theta_{1MN}^{0(2)} \\
 & \lambda_{11}^{1(1)}, \theta_{11}^{1(1)} & \lambda_{11M}^{1(2)}, \theta_{11M}^{1(2)} \\
 & & \lambda_{111}^{2(2)}, \theta_{111}^{2(2)}
\end{array}
$$

Figure 2: Reducible λ and θ fields for the $D = 5$ BF theory , $M, N = 1, 2, 3$

In the matrix like structure shown in figure 2 each row defines two independent tree diagrams (one for the λ and one for the θ fields) like those ones defined in figure 1. In each field the label appearing before the bracketed superscript tracks the level in which each tree begins while as stated before, the bracketed superscript defines the stage of

reducibility to which the object does actually belong. For example, $\lambda^{1(2)}_{11M}$ is a second stage auxiliary field belonging to a tree begining at the first stage of reducibility. Notice that the arrengement is upper triangular, meaning that there are no ghosts either for $\lambda^{2(2)}_{111}$ or $\theta^{2(2)}_{111}$.

All these auxiliary fields are needed to ensure both the off-shell nilpotency of the BRST transformations and the construction of a BRST invariant effective action. The capital subscripts (identifiers) that appear in each field are related to the role of each object. Indeed, and as one may observe in the effective action to be built below, any field whose last subscript is 1 behaves as a ghost associated to some of the gauge symmetries of the original action, if the last identifier is a 2 the object is an antighost while if it is 3 the field is a Lagrange multiplier accompanying a gauge fixing condition.

Let us now turn our attention to the construction of the effective action. Since the theory under study has a vanishing Hamiltonian its effective action is given by [10]:

$$\begin{aligned} S_{eff} = \int_{ti}^{tf} dt[&\pi^i \dot{A}_i + \mu^1 \dot{C}_1 + \mu^{1ij}_{(0)} \dot{C}^{(0)}_{1ij} + \mu^{11j}_{(1)} \dot{C}^{(1)}_{11j} + \mu^{111}_{(2)} \dot{C}^{(2)}_{111} \\ & \widehat{\delta}(\lambda_1 \mu^1 + \lambda^{0(0)}_{1ij} \mu^{1ij}_{(0)} + \lambda^{1(1)}_{11i} \mu^{11i}_{(1)} + \lambda^{2(2)}_{111} \mu^{111}_{(2)}) + L_{GF+FP}], \end{aligned} \tag{14}$$

where

$$\begin{aligned} L_{GF+FP} =& \widehat{\delta}(C_2 \chi_2 + C^{(0)}_2 \chi^{(0)}_2) + \widehat{\delta}(\sum_{M=1}^{3} C^{(1)}_{M2} \chi^{(1)}_{M2} + \sum_{M,N=1}^{3} C^{(2)}_{MN2} \chi^{(2)}_{MN2}) + \\ & \widehat{\delta}(\lambda^{0(1)}_{12} \Lambda^{0(1)}_2 + \sum_{M=1}^{3} \lambda^{0(2)}_{1M2} \Lambda^{0(2)}_{M2} + \lambda^{1(2)}_{112} \Lambda^{1(2)}_2) + \\ & \widehat{\delta}(\theta^{0(1)}_{12} \Theta^{0(1)}_2 + \sum_{M=1}^{3} \theta^{0(2)}_{1M2} \Theta^{0(2)}_{M2} + \theta^{1(2)}_{112} \Theta^{1(2)}_2), \end{aligned} \tag{15}$$

is the sum of the generalizations of the Fadeev-Popov and gauge fixing terms. In (15) χ_2, $\chi^{(0)}_2$ are the primary gauge fixing functions associated to the constraints (4a) and (4b), while $\chi^{(1)}_{M2}$, $\chi^{(2)}_{MN2}$, $\Lambda^{0(1)}_2$, $\Lambda^{0(2)}_{M2}$, $\Lambda^{1(2)}_2$ $\Theta^{0(1)}_2$,$\Theta^{0(2)}_{M2}$ and $\Theta^{1(2)}_2$ are gauge fixing functions which must fix the longitudinal part of the fields in the non minimal sector. As usual, the BRST transformation for the canonical variables (Z) is given by

$$\widehat{\delta} Z = (-1)^{\epsilon_z} \{Z, \Omega\}, \tag{16}$$

where ϵ_z is the grassmanian parity of Z, while the BRST transformation of the variables belonging to the non minimal sector are fixed by imposing the closure of the charge as discussed in [10]. Their explicit form for this system may be read from the expressions we give below for the general D-dimensional case.

Let us now turn to the general case. For D$\geq$ 6 once again the action (1) can be written as a constrained canonical action with vanishing Hamiltonian:

$$S = \int d^D x (\dot{A}^a_i \pi^{ia} + A^a_0 \phi^a + B^a_{0 i_1 \cdots i_{D-1}} \Phi^{i_1 \cdots i_{D-1} a}). \tag{17}$$

This time the constraints are defined as follows

$$\phi^a \equiv (\nabla_i \pi^i)^a = \partial_i \pi^{ia} + (A_i \times \pi^i)^a = \partial_i \pi^{ia} + f^{abc} A_i^b \pi^{ic} = 0, \tag{18a}$$

$$\Phi^{i_1 \cdots i_{D-1} a} \equiv \epsilon^{i_1 \cdots i_{D-1}} (F^a_{i_{D-2} i_{D-1}}) = 0. \tag{18b}$$

Clearly, the constraints have the same structure seen in the 4 [9] and 5 dimensional cases; in fact, the ϕ^a constraints are exactly the same. The Poisson Bracket algebra of these constraints is still first class and explicitly given by

$$\{\phi^a(x), \phi^b(x')\} = f^{abc} \phi^c(x) \delta^{D-1}(x - x'), \tag{19a}$$

$$\{\Phi^{i_1 \cdots i_{D-1} a}(x), \Phi^{j_1 \cdots j_{D-1} b}(x')\} = 0, \tag{19b}$$

$$\{\phi^a(x), \Phi^{i_1 \cdots i_{D-1} a}(x')\} = f^{abc} \Phi^{i_1 \cdots i_{D-1} a}(x) \delta^{D-1}(x - x'). \tag{19c}$$

As mentioned the D-dimensional BF theories have D-3 stages of reducibility which are defined through the following operators

$$a^{(1)} \equiv a^{(1) j_1 \cdots j_{D-4} a}_{i_1 \cdots i_{D-3}} = \delta^{j_1}_{[i_1} \delta^{j_2}_{i_2} \cdots \delta^{j_{D-4}}_{i_{D-4}} \nabla^a_{i_{D-3}]}, \tag{20a}$$

$$a^{(2)} \equiv a^{(2) j_1 \cdots j_{D-5} a}_{i_1 \cdots i_{D-4}} = \delta^{j_1}_{[i_1} \delta^{j_2}_{i_2} \cdots \delta^{j_{D-5}}_{i_{D-5}} \nabla^a_{i_{D-4}]}, \tag{20b}$$

$$\cdots$$

$$a^{(D-4)} \equiv a^{(D-4) ka}_{ij} = \delta^k_{[i} \nabla^a_{j]}, \tag{20c}$$

$$a^{(D-3)} \equiv a^{(D-3) a}_m = \nabla^a_m. \tag{20d}$$

For these theories [10] the minimal sector of the extended phase space is itself divided in two parts: one composed by the fields associated to the irreducible constraints which are the same as in the lower dimensional case, see (9b), and one composed by the fields associated to the reducible constraints which are the following canonical pairs that generalize (9c) (notice the geometric structure given by the space-like indices):

$$\begin{aligned} &(C^{(0)}_{1 i_1 \ldots i_{D-3}}, \mu^{1 i_1 \ldots i_{D-3}}_{(0)}) \\ &(C^{(1)}_{11 i_1 \ldots i_{D-4}}, \mu^{11 i_1 \ldots i_{D-4}}_{(1)}) \end{aligned} \tag{21a}$$

$$\cdots$$

$$(C^{(p-1)}_{\underbrace{1 \ldots 1}_{p} i_1 \ldots i_{D-2-p}}, \mu^{\overbrace{1 \ldots 1}^{p} i_1 \ldots i_{D-2-p}}_{(p-1)}) \tag{21b}$$

$$\cdots$$

$$(C^{(D-3)}_{\underbrace{1 \ldots 1}_{D-2}}, \mu^{\overbrace{1 \ldots 1}^{D-2}}_{(D-3)}). \tag{21c}$$

To simplify the reading we introduce the following compact notation to be used hereon: a bracketed subscript (or superscript) will refer to the number of "ones" which label an object while the $(D-2-p)$ space indices will be represented by a greek multi-index. For example:

$$C_{[p]\epsilon} \equiv C^{(p-1)}_{\underbrace{1\ldots1}_{p} i_1\ldots i_{D-2-p}}$$

$$\mu^{[p]\epsilon} \equiv \mu_{(p-1)}^{\overbrace{1\ldots1}^{p} i_1\cdots_{D-2-p}}, \ p=1,2,\ldots,D-2. \tag{22}$$

Additionaly we also use the following concise notation for the reducibility operators,

$$a^{(p)} \equiv a^{(p)\epsilon}_{\nu} \equiv \underbrace{\delta^{j_1}_{[i_1}\delta^{j_2}_{i_2}\cdots\delta^{j_{D-2-p}}_{i_{D-2-p}}}_{(p-1) delta factors}\nabla^a_{i_{D-1-p}]}, \ p=1,2,\ldots,D-3. \tag{23}$$

With this notation the general expression for the off shell nilpotent BRST operator is given by

$$\begin{aligned}\Omega^{\mathcal{D}} =& < C_1\phi + C_{[1]\mu}\Phi^{\mu} + \sum_{p=2}^{\mathcal{D}-2} C_{[p]\mu}[\delta\nabla]^{\mu}_{\nu}\mu^{[p-1]\nu}) \\ & - \frac{1}{2}C_1(C_1\times\mu^1) - \sum_{p=1}^{\mathcal{D}-2} C_{[p]\mu}(C_1\times\mu^{[p]\mu}) \\ & + \sum_{p+q+1=3}^{p+q+1=\mathcal{D}-2}(2-\delta_{pq})A_{pq}C_{[p+q+1]\mu}(\mu^{[p]\mu\nu}\times\mu^{[q]\alpha})\epsilon_{\nu\alpha} > .\end{aligned} \tag{24}$$

This expression for the BRST charge has essentially the same structure that we observed in the five dimensional case. In fact we recognize the Yang Mills like terms, the $C_{[p]}[\delta\nabla]\mu^{[p-1]}$ terms and the new polinomial $C_{[p+q+1]}(\mu^{[p]}\times\mu^{[q]})$ terms that ensure the off-shell nilpotency of the charge. The unknown coefficients can be explicitly calculated through a recurrence relation which for $D\geq 5$ and $p\geq q$ yields:

$$(D-2-p-q)!A_{p,q-1} + (-1)^q(D-2-p)!A_{p+1,q-1} = 0, \tag{25a}$$

$$(-1)^{p+1}(D-2-p-q)!A_{p,q-1} + (-1)^{D-1}(q+1)(D-2-q)!A_{p,q} = 0, \tag{25b}$$

$$A_{D-4,1} = \frac{(-1)^D}{2(D-3)!}, \tag{25c}$$

$$A_{1,1} = -\frac{(D-4)!}{4}. \tag{25d}$$

It only remains to construct the non minimal sector of the phase space, the BRST transformation properties of their fields and the effective action. The fields in the

minimal sector associated to the irreducible constraints are those given by Table 1. To display the complete set of fields in the non-minimal sector associated to the reducible constraints one has to generalize the tree diagrams of figures 1 and 2. The non minimal C fields are given by:

$$C^{(p)}{}_{\underbrace{SJ\cdots I}_{(p+1)-subscripts}} \qquad S, J, ..., I = 1, 2, 3; \quad p = 0, 1, \ldots, D-3 \tag{26}$$

where at least one of the capital subscripts must take the values 2 or 3.

And the λ and θ fields which complete the set of local coordinates of the extended phase space:

$$\lambda^{r(s)}_{[r+1]\underbrace{M\cdots Q}_{s-r}\epsilon} \equiv \lambda^{r(s)}_{\underbrace{1\cdots 1}_{r+1}\underbrace{M\cdots Q}_{s-r}\epsilon} \tag{27a}$$

$$\theta^{r(s)}_{[r+1]\underbrace{M\cdots Q}_{s-r}\epsilon} \equiv \theta^{r(s)}_{\underbrace{1\cdots 1}_{r+1}\underbrace{M\cdots Q}_{s-r}\epsilon} \tag{27b}$$

$r = 0, 1, \ldots, D-3; s = 0, \ldots, D-3-r;\; M, Q, P = 1, 2, 3$

Next we define the BRST transformation rules for all the fields. In the minimal sector, the BRST transformation is simply given by:

$$\widehat{\delta} Z = (-1)^{\epsilon_z}\{Z, \Omega\}. \tag{28}$$

In the non-minimal sector the BRST transformation laws must be calculated to ensure their off-shell nilpotency (i.e. $\widehat{\delta}\widehat{\delta}$(anything)=0). To achieve this goal we need to introduce the transverse-longitudinal (T+L) decomposition of geometrical objects with respect to the reducibility operators ($a^{(p)\epsilon}_{\mu}$).

Given a lower multi-indexed geometrical object V_ϵ its T+L decomposition is given as:

$$V_\epsilon = V^T_\epsilon + a^{(p)\mu}_\epsilon V_\mu \tag{29a}$$

$$A^{(p)\epsilon}_\mu V^T_\epsilon = 0 \tag{29b}$$

$$V^L_\mu = A^{(p)\epsilon}_\mu V_\epsilon. \tag{29c}$$

Similarly, for upper multi-indexed objects (V^ρ), the T+L decomposition is defined through

$$V^\rho = V^{T\rho} + A^{(p)\rho}_\mu V^{L\mu} \tag{30a}$$

$$a^{(p)\mu}_\rho V^{T\rho} = 0 \tag{30b}$$

$$V^{L\rho} = a^{(p)\rho}_\mu V^\mu \;\; p = 1, \ldots, D-3. \tag{30c}$$

It is important to remark that there is always possible to find a set of operators $A^{(p)\mu}_\nu (p = 1, \ldots, D-3)$ such that the above decompositions are unique. Next, let

t stand for any variable of the non-minimal sector. The BRST transformation rules are given by

$$\widehat{\delta}t^{(i+j)}_{\underbrace{M\cdots Q}_{i-1}2[j]\epsilon} = t^{(i+j)}_{\underbrace{M\cdots Q}_{i-1}3[j]\epsilon} + a^{(i+j+1)\mu}_{\epsilon}t^{(i+j+1)}_{\underbrace{M\cdots Q}_{i-1}2[j+1]\mu} \tag{31a}$$

$$\widehat{\delta}t^{(i+j)}_{\underbrace{M\cdots Q}_{i-1}3[j]\epsilon} = -\widehat{\delta}a^{(i+j+1)\mu}_{\epsilon}t^{(i+j+1)}_{\underbrace{M\cdots Q}_{i-1}2[j+1]\mu} - a^{(i+j+1)\mu}_{\epsilon}\widehat{\delta}t^{(i+j+1)}_{\underbrace{M\cdots Q}_{i-1}2[j+1]\mu} \tag{31b}$$

for $1 \leq i \leq D-3; 1 \leq i+j \leq D-4$ (If the object is upper indexed one must change the $a^{(p)\mu}_{\nu}$ operator for the $A^{(p)\mu}_{\nu}$ one).

For the last level of reducibility one finds

$$\widehat{\delta}t^{(D-3)}_{\underbrace{M\cdots Q}_{D-3}2\epsilon} = t^{(D-3)}_{\underbrace{M\cdots Q}_{D-3}3\epsilon} \tag{31c}$$

$$\widehat{\delta}t^{(D-3)}_{\underbrace{M\cdots Q}_{D-3}3\epsilon} = 0 \tag{31d}$$

$$\widehat{\delta}t^{(D-3)}_{\underbrace{M\cdots Q}_{i}2[j]\epsilon} = t^{(D-3)}_{\underbrace{M\cdots Q}_{i}2[j]\epsilon} \tag{31e}$$

$$\widehat{\delta}t^{(D-3)}_{\underbrace{M\cdots Q}_{i}3[j]\epsilon} = 0;\ i+j = D-3. \tag{31f}$$

For the λ and θ fields which define the begining of a tree diagram the transformation laws are

$$\widehat{\delta}\lambda^{i(i)}_{[i+1]\mu} = \theta^{i(i)}_{[i+1]\mu} + a^{(i+1)\rho}_{\mu}\lambda^{i(i+1)}_{[i+2]\rho} \tag{32a}$$

$$\widehat{\delta}\theta^{i(i)}_{[i+1]\mu} = -\widehat{\delta}a^{(i+1)\rho}_{\mu}\lambda^{i(i+1)}_{[i+2]\rho} - a^{(i+1)\rho}_{\mu}\widehat{\delta}\lambda^{i(i+1)}_{[i+2]\rho} \tag{32b}$$

$$i = 0, \ldots D-4$$

and finally:

$$\widehat{\delta}\lambda^{D-3(D-3)}_{[D-2]} = \theta^{D-3(D-3)}_{[D-2]} \tag{32c}$$

$$\widehat{\delta}\theta^{D-3(D-3)}_{[D-2]} = 0. \tag{32d}$$

With the BRST transformation rules given above, the effective action is built as a generalization of (14) as follows [10]:

$$Seff = \int_{ti}^{tf} dt[\pi^i\dot{A}_i + \mu^1\dot{C}_1 + \sum_{p=0}^{D-3}\mu^{[p+1]}\dot{C}_{[p+1]} + \widehat{\delta}(\sum_{p=0}^{D-3}\lambda^{p(p)}_{[p+1]}\mu^{[p+1]}) + L_{GF+FP}]. \tag{33}$$

this time the Fadeev-Popov + gauge fixing Lagrangian is given by a longer expression which includes enough terms as to completely fix the gauge associated to the reducible constraints.

$$
\begin{aligned}
L_{GF+FP} =& \widehat{\delta}(C_2\chi_2 + C_2^{(0)}\chi_2^{(0)} + \\
& \widehat{\delta}(\sum_{M=1}^{3} C_{M2}^{(1)}\chi_{M2}^{(1)} + \cdots + \sum_{M,N,\ldots P=1}^{3} C^{(D-3)}_{\underbrace{MN\ldots P}_{D-2}2}\chi^{(D-3)}_{MN\ldots P2}) \\
& \widehat{\delta}(\sum_{r=0}^{D-4}\sum_{s=r+1}^{D-3}\sum_{M,N,\ldots,P=1}^{3} \lambda^{r(s)}_{[r+1]\underbrace{MN\ldots P}_{s-r-1}2}\Lambda^{r(s)}_{\underbrace{MN\ldots P}_{s-r-1}2}) + \\
& \widehat{\delta}(\sum_{r=0}^{D-4}\sum_{s=r+1}^{D-3}\sum_{M,N,\ldots,P=1}^{3} \theta^{r(s)}_{[r+1]\underbrace{MN\ldots P}_{s-r-1}2}\Theta^{r(s)}_{\underbrace{MN\ldots P}_{s-r-1}2}) \\
& .
\end{aligned}
\tag{34}
$$

The admissible set of gauge choices includes only the ones which fix the longitudinal part of the associated fields. Within this set the modified BFV approach [10-11] ensures that the functional integral of the theory (defined as the sum over histories of $exp(-S_{eff})$ with unit weight) is locally independent of the gauge choice provided the following boundary condition holds:

$$
[\Omega - \pi^{\rho}\frac{\partial\Omega}{\partial\pi^{\rho}} - \sum_{p=1}^{D-3}\mu^{[1]\rho}\frac{\partial\Omega}{\partial\mu^{[1]\rho}}]\ |_{t_{in}}^{t_{fin}} = 0. \tag{35}
$$

From the obtained expression for the BRST charge Ω one verifies that the partition function Z is **independent of the gauge coupling constant** (e^2). In fact the effective action may be written as a linear homogeneous quantity in the following set of variables:

$$
\pi^i, \mu, C_{[1]}, C_{[2]}, \ldots, C_{[D-2]} \tag{36a}
$$

and

$$
\lambda^{r(p)}_{[p+1]}, \theta^{r(p)}_{[p+1]},\ p = 0, \ldots, D-3\ , r = 0, \ldots, D-3 \tag{36b}
$$

plus the gauge fixing terms. The gauge coupling constant may then be absorbed by redefining the set (36) together with a change in the gauge fixing choice provided $e^2 \neq 0$. Since the partition function Z is independent of the gauge fixing condition, Z does not depend on the coupling constant. This property allows Z to be evaluated by going to the limit of very small e^2 where the path integral is dominated by the classical minima. We may then conclude that the semi-classical limit is exact.

The independence on the coupling constant is a common link between many topological gauge theories (see [13],[16],[18]). For Witten's topological theory it was proved in [13] using the fact that the Lagrangean for such theory is of the form $\{\Omega, \Psi\}$. In the case of the non abelian BF theories the effective Lagrangean has not this structure

[9] but our argument based on the form of the Ω operator allows to reach the same conclusion.

REFERENCES

[1] G. T. Horowitz, *Commun. Math.Phys.***125** (1989) 417.
[2] G. T. Horowitz and M. Srednicki, *Commun. Math.Phys.***130** (1990) 83.
[3] M. Blau and G. Thompson, *Ann.Phys.* **205** (1991) 130; D. Birmingham, M. Blau, M. Rakowski and G. Thompson, *Phys. Rep.* **209** (1991) 129.
[4] B. Broda, *Phys. Lett.* **B254** (1991) 111.B. Broda, *Phys. Lett.* **B280** (1992) 47.
[5] M. Blau and G. Thompson, *Phys. Lett.* **255** (1991) 535.
[6] M. Abud and G. Fiore,*Triestre Preprint SISSA 159/91 EP and Napoli Preprint INFN, NA IV 21/91.*
[7] J.C. Wallet, *Phys. Lett.* **235** (1990) 71.
[8] C. Lucchesi, O. Piguet and S.P. Sorella, *Nucl. Phys.* **B395** (1993) 325.
[9] M. I. Caicedo and A. Restuccia, *Phys. Lett.* **B307** (1993) 77.
[10] M. I. Caicedo and A. Restuccia, *Class. Quan. Grav* **10** (1993) 833.
[11] M. I. Caicedo, *Doctoral Thesis, Universidad Simón Bolívar* (1993); Preprint **SB/F/93-221**.
[12] E. S. Fradkin and G. A. Vilkovisky, *Phys. Lett.* **B55** (1975) 224; CERN report TH-2332, (1977); I. Batalin and E. Fradkin, *Phys. Lett.* **B122** (1983) 157; *Phys. Lett.* **B128** (1983) 307; *Ann Inst Henri Poincaré* **49** (1988) 215.
[13] E. Witten,*Commun. Math. Phys.* **117** (1988) 353.
[14] J. M. F. Labastida and M. Pernici, *Phys. Lett.* **B212** (1988) 56; F.De Jonghe and S.Vandoren, *Phys. Lett.* **B324** 328.
[15] L. Baulieu and I. M. Singer, *Nucl. Phys.* (Proc. Suppl.) **5B** (1988) 12; Y. Igarashi, H. Imai, S. Kitakado and H. So, *Phys. Lett.* **B227** (1989) 239; C. Aragão and L. Baulieu, *Phys. Lett.* **B275** (1992) 315.
[16] R.Gianvittorio, A.Restuccia and J.Stephany, Preprint **SB/F/94-225** *Phys Lett.B* to appear (hep-th/9410123).
[17] N. Seiberg and E. Witten, *hep-th/9407087, hep-th/9408099.*
[18] L.F.Cugliandolo,G.Lozano and F.A.Schaposnik, *Phys.Lett.* **B234** (1990) 52.